

Problem 8

Use equation (1.8) to find the fractions that are equivalent to the following repeating decimals:

$$0.694444 \dots$$

Solution

$$\begin{aligned} 0.694444 \dots &= 0.69 + 0.004 + 0.0004 + \dots \\ &= \frac{69}{100} + \frac{4}{1000} + \frac{4}{10\,000} + \dots \\ &= \frac{69}{100} + \sum_{i=0}^{\infty} \left(\frac{4}{1000} \right) \frac{1}{10^i} \\ &= \frac{69}{100} + \sum_{i=0}^{\infty} \left(\frac{4}{1000} \right) \left(\frac{1}{10} \right)^i \\ &= \frac{69}{100} + \frac{\frac{4}{1000}}{1 - \left(\frac{1}{10} \right)} \\ &= \frac{25}{36} \end{aligned}$$